The Tenuous Relationship between Effort and Performance Pay*

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Abstract

When a worker is offered performance related pay, the incentive effect is not only determined by the shape of the incentive contract, but also by the probability of contract enforcement. We show that weaker enforcement may reduce the worker’s effort, but lead to higher-powered incentive contracts. This creates a seemingly negative relationship between effort and performance pay.

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1 Introduction

Standard economic models predict a positive relationship between effort and performance pay. In contrast, there is a range of sociological and psychological studies that focus on all the problems that performance pay creates. Some studies even suggest that performance pay can be detrimental to effort (see Jenkins, Gupta, Mitra and Shaw, 1998, for an overview). The negative effects of so-called New Public Management (NPM) are often emphasized. NPM describes reforms in the public sector that are characterized by an emphasis on output control, performance related pay and introduction of market mechanisms. Scholars argue that NPM undermines - or crowd out - intrinsic motivation and thus the effort of public servants, see e.g. Weibel, Rost, Osterloh (2010), and Perry, Engbers and Jun (2009).

This incentive puzzle has gained inquisitive interest from economic theorists. The common denominator of the different theoretical approaches is that non-monetary motivation is treated as a variable as opposed to a fixed attribute.¹ Standard economic theory acknowledges that agents have non-monetary motivation, but it is treated as a fixed entity. Once non-monetary motivation is allowed to vary, higher monetary rewards may reduce non-monetary motivation to such an extent that effort is reduced.

In this paper we show that variations in enforcement probability can have similar effects as variations in intrinsic motivation, and we argue that the former can be an alternative explanation for a negative association between performance pay and effort. If there is a probability \( v \in (0, 1) \) that an incentive contract is enforced, and this probability is treated as a variable rather than as a fixed parameter, then higher monetary rewards in the incentive contract may be associated with a lower probability of enforcement.

¹Recent papers show how the structure of monetary rewards may undermine incentives for social esteem (Benabou and Tirole, 2006, and Ellingsen and Johannesson, 2008), affect agents' internal rewards from norm adherence (Sliwka, 2007), or affect agents' perception of their tasks or own abilities (Benabou and Tirole, 2003). See Frey and Regel (2001) for a review of previous literature on motivation crowding out.
This may lead to reduced effort.

We concentrate on informal contract enforcement. Informal enforcement is often modelled as a repeated game where contract breach is punished, not by the court, but by the contracting parties themselves who can refuse to cooperate or trade with each other after a deviation. But informal enforcement can also be due to moral or social commitment. Greif (1994) defines moral enforcement as enforcement based on the tendency of humans to derive utility from acting according to their values, while social enforcement is related to social sanctions. In this paper, we assume that contracts are not enforceable by the court of law, but that there is a probability \( v \in (0, 1) \) that the principal feels morally or socially committed to honor the contract. Moreover, we assume that the principal learns whether or not she will actually honor the contract after the contract is offered. There are two possible justifications for this. One is that the principal may learn ex post about the contractual environment, for instance to which extent social or reputational concerns matter for the given contractual relationship. Another possibility is that the principal learns about her own type after observing her own contract offer and the agent’s actions.

Now, why should improved enforcement, i.e. higher probability that the principal honors the contract, lead to lower-powered incentive contracts? At the outset one might expect the opposite. No incentive contract can be implemented in a situation where the principal certainly won’t pay. And high-powered incentives can certainly be enforced if the contract is honored for sure. Also, risk aversion on the part of the agent can make it quite costly for the principal to offer incentives where very high bonuses are paid with low probability, as the agent must be compensated for the high risk associated with such schemes. However, it turns out that on the margin, the incentive intensity of the contract can be negatively related to the probability of enforcement under quite standard assumptions.

We show this in a simple moral hazard model where a principal must
provide an agent with incentives to exert effort, and where the incentive contract is honored with a probability $v$. We deduce the optimal incentive contract and study how exogenous variations in $v$ affect incentive provision. Exogenous variations in informal contract enforcement occur across countries and industries, but can also affect a given contractual relationship via organizational or institutional reforms. Both the environments for reputational enforcement, and the conditions for social and moral commitment may vary. As an example of the latter, it is shown in several experiments that communication facilitates trust and trustworthiness. In particular it is shown that stated promises increases the likelihood of trustworthy behavior (Ellingsen and Johannesson, 2004, Charness and Dufwenberg, 2006). One would thus expect stronger moral enforcement in environments where the principal can easily communicate with the agent.

We first adopt the classical model on risk sharing vs. incentives (e.g. Holmström 1979), and show that when enforcement is probabilistic, then under certain conditions contractual incentive intensity and effort are negatively related. We then show that a similar result can also be obtained under risk neutrality and limited liability. This negative relationship is a "false crowding out effect" since total monetary incentives, which is the product of the enforcement probability and contractual incentives, is positively related to effort. But since the enforcement probability does not show up in the incentive contract, it appears that incentives and effort are negatively related.

To see the intuition, note that if the enforcement probability increases, this has a positive effect on effort, but it also increases expected wage costs per unit of effort since the probability that the principal actually has to pay as promised increases. In order to reduce wage costs, the principal can simply reduce expected contractual wage payments. Hence, effort increases, but the contractual incentives are lower-powered. And the other way around: Weaker enforcement induces lower effort since the probability that the agent actually
is paid decreases. In order to mitigate the reduction in effort, the principal can thus provide higher-powered incentives.

This result has an important empirical implication: When observing a negative relationship between performance pay and effort, one has to control for the probability that incentive contracts are actually honored. If not, one may wrongfully infer that monetary incentives crowd out non-monetary motivation. Controlling for enforcement probability is quite easy in experimental work.\(^2\) In empirical work, however, this is much more of a challenge. Take New Public Management (NPM) as an example. As noted, many scholars argue that NPM undermines intrinsically motivated effort. But if NPM actually undermines effort (which of course is debatable, see Stazyk, 2010), would this necessarily come from crowding out of intrinsic motivation? Important aims of NPM include decentralization of management authority, more discretion and flexibility, less bureaucracy and less rules. These institutional changes may affect both the legal and the informal enforcement environment.

The crux is that enforcement and contractual incentives may be substitutes. In that sense our paper is related to models showing the substitutability between explicit contracts and informal relational contracts (see Baker, Gibbons and Murphy, 1994, and Schmidt and Schnitzer, 1995). In these models, improved explicit contracts may reduce feasible incentive pay under relational contracting, but effort is still positively related to the sum of contractual incentives. In contrast, we find that effort may be negatively related to contractual incentives.

With respect to the modelling, a contribution of the paper is to consider probabilistic enforcement in an otherwise standard moral hazard model with risk aversion or limited liability. In the classic moral hazard models (e.g. Holmström, 1979), perfect enforcement is assumed, while in models of incomplete contracting, it is commonly assumed that contracting is prohibitively

\(^2\)There are a few of laboratory and field experiments documenting a negative causal relationship between effort and monetary incentives (e.g. Frey and Oberholzer-Gee, 1997; Gneezy and Rustichini, 2000, and Fehr and Gachter, 2002).
costly so that legal enforcement is impossible (starting with Grossman and Hart, 1986).\footnote{However, imperfect enforcement is increasingly recognized as an important ingredient in models of contractual relationships. Some papers focus on the relationship between ex post evidence disclosure and enforceability (Ishiguro, 2002; Bull and Joel Watson, 2004), while others focus on the relationship between ex ante contracting and enforceability (Battigalli and Maggi, 2002, Schwartz and Watson, 2004, Shavell 2006). There is also a growing literature on the interaction between legal imperfect enforcement and informal (relational) enforcement, see Sobel (2006), MacLeod (2007), Battigalli and Maggi (2008) and Kvaløy and Olsen (2009, 2012).} The way we model probabilistic enforcement is also novel. We adopt the general idea from the incomplete contracting literature that necessary information is realized ex post. In the seminal papers by Grossman and Hart (1986) and Hart and More (1990), the principal (buyer) learns about her needs ex post. In our setting, the principal learns about the contractual environment or her own type ex post. \footnote{The latter bears some resemblance to the literature on will-power and self-control where people learn about their own type from previous actions (see e.g. Benabou and Tirole, 2003)}

The paper is organized as follows. In Section 2 we present the basic model. In Section 3 we study variations in enforcement probability under risk aversion and limited liability, respectively. Section 4 concludes.

\section{Model}

We consider a relationship between a principal and an agent, where the agent produces output $x$ for the principal. Output is a random variable ($x \in X$), and the agent’s effort $a$ affects the probability distribution (density) $f(x,a)$. Effort costs are given by $C(a)$, where $C'(a) > 0$, $C''(a) > 0$, $C(0) = 0$. We assume that output is observable to both parties, but that the agent’s effort level is unobservable to the principal, so the parties must contract on output: the principal pays a wage $w(x) = s + \beta(x)$ where $s$ is a non-contingent fixed salary and $\beta(x)$ is a contingent bonus ($\beta(x) < 0$ implies a contingent fine). We assume that the principal is risk neutral, but allow the agent to be risk
averse, with a utility function $u(w)$.

We assume that contracts are not enforceable by the court of law, but that there is a probability $v \in (0, 1)$ that the principal feels (morally or socially) committed to honor the contract. Consider then the following stage game $\Gamma$:

1. The principal offers a contract $w(x) = s + \beta(x)$ to the agent. If the agent rejects the offer, the game ends. If he accepts, the game continues to stage 2.

2. The agent takes action $a$ and realizes output $x$.

3. Nature draws. With probability $v$ the principal finds herself committed to pay the bonus $\beta(x)$.

4. The principal observes $x$, pays $s$ and chooses bonus payment $\tilde{\beta}(x) = \beta(x)$ if she is committed to honor the contract, and $\tilde{\beta}(x) = 0$ if not.

A crucial assumption here is that the principal learns whether or not she will actually honor the contract after the contract is offered. As noted, there are two possible justifications for this. One is that the principal may learn about the contractual environment in stage 3, for instance to which extent social or reputational concerns matter for the given contractual relationship. Another possibility is that there are two types of principals, one that honors and one that reneges on promises, and the principal learns about her own type in stage 3 of the game.

### 3 Incentives and enforceability

We will now deduce the optimal contract and discuss variations in enforcement probability $v$. We will first assume that the agent is risk averse. We will then analyze the case where both parties are risk neutral but subject to limited liability.
3.1 Risk aversion

In stage 2 the game $\Gamma$, the agent chooses effort to maximize his expected utility, given by

$$U(a, w, v, s) = v \int f(x, a)u(w(x))dx + (1 - v)u(s) - C(a).$$

(Unless otherwise noted, all integrals are over the support $X$.) For each outcome $x$, the agent gets the payment $w(x) = s + \beta(x)$ with probability $v$, and the payment (fixed salary) $s$ otherwise, and this gives expected utility as specified. Optimal effort satisfies

$$U_a(a, w, v, s) = v \int f_a(x, a)u(w(x))dx - C'(a) = 0$$

(IC)

(We will invoke assumptions to make the 'first-order approach' valid.)

In stage 1 the principal chooses wages (and effort $a$) to maximize her payoff, subject to the agent’s choice, represented by IC, and the agent’s participation constraint:

$$U(a, w, v, s) \geq U_o$$

(IR)

The principal, assumed risk neutral, has payoff

$$V(a, w, v, s) = \int f(x, a) [x - vw(x)] dx - (1 - v)s$$

Forming the Lagrangian $L = V + \lambda(U - U_o) + \mu U_a$, with multipliers $\lambda$ and $\mu$ on the IR and IC constraints, respectively, one sees that optimal payments satisfy

$$\frac{1}{u'(w(x))} = \lambda + \mu \frac{f_a(x, a)}{f(x, a)}, \quad \frac{1}{u'(s)} = \lambda$$

(W)

These conditions are standard (Holmström 79), and reflect the trade-off between providing insurance and incentives for the agent. This trade off is relevant for the performance dependent bonuses, but not for the fixed pay-
ment $s$. Given a monotone likelihood ratio $\frac{f_{a}(x,a)}{f(x,a)}$ (MLRP), payments $w(x)$ will be increasing in output $x$.

Payments will be chosen to implement the action that is optimal for the principal, and this entails an action that satisfies $L = 0$. The optimal action and the associated payments (and multipliers) will depend on the parameter $v$, i.e. on the level of enforceability.

We now ask, i) will effort increase when the enforcement probability $v$ increases and ii) may contractual incentives at the same time become weaker? That is: would the new contractual incentives (corresponding to the higher $v$) have induced lower effort under the old $v$? If so, the new contractual incentives are weaker, but the associated effort will be higher.

Consider the agent’s (marginal) incentives for effort; they are given by $vm(a, w)$, where

$$m(a, w) \equiv \int f_{a}(x,a)u(w(x))dx$$

(M)

Thus $m(a, w)$ is the marginal incentive for effort generated by the contract $w(x) = s + \beta(x)$. We call $m$ the marginal contractual incentives.

Consider now $\tilde{v} > v$, and suppose the associated optimal efforts satisfy $\tilde{a} > a$. A way to interpret question ii) is then to ask whether $m(a, \tilde{w}) < m(a, w)$, i.e. whether the monetary payments $\tilde{w}$ associated with the higher $\tilde{v}$ yield in isolation lower marginal incentives for the agent.

Now, optimal effort and payments are functions of $v$, say $a(v)$ and (with some abuse of notation) $w(v)$, respectively. We thus ask if $m(a, w(v))$ is decreasing in $v$, i.e. if

$$\frac{\partial}{\partial v}m(a, w(v)) = \int f_{a}(x,a)\frac{\partial}{\partial v}u(w(x; v))dx < 0$$

Note that in equilibrium the agent’s choice of effort will be $a = a(v)$, and hence we have from incentive compatibility (IC) that $vm(a(v), w(v)) = C'(a(v))$. Differentiating this identity we see that for equilibrium effort $a =
For $a(v)$ we have

$$v \frac{\partial}{\partial v} m(a, w(v)) = \left[ C''(a) - v \frac{\partial}{\partial a} m(a, w(v)) \right] a'(v) - C'(a)/v$$  \hspace{1cm} (1)$$

From this it follows that if $a'(v) > 0$ (so effort increases with $v$), and the last term dominates the other terms on the RHS (so $\frac{\partial}{\partial v} m < 0$), then it will be the case that effort and marginal contractual incentives for effort move in opposite directions.\footnote{If on the other hand $a'(v) < 0$, then (since the square bracket in (1) is positive by the agent's SOC), we will have $\partial m/\partial v < 0$, and thus effort and marginal contractual incentives moving in the same direction.} We will in the following provide a specification of functional forms where this is precisely the case.

Note from (1) and IC ($vm = C'$) that the sign of $\frac{\partial}{\partial v} m$ is given by the sign of

$$\left[ \frac{aC''(a)}{C'(a)} - \frac{a}{m(a, w)} \frac{\partial}{\partial a} m(a, w) \right] \frac{va'(v)}{a} - 1$$  \hspace{1cm} (2)$$

Hence the sign is determined by the magnitudes of three elasticities; pertaining to marginal costs, marginal contractual incentives and equilibrium effort, respectively. Signing expressions like (1) thus requires properties of equilibrium effort variations in a moral hazard model. To make this tractable we consider specific functional forms. Assume the following specifications for the probability distribution and for the agent’s utility:

$$F(x, a) = \Pr(\text{outcome} \leq x \mid a) = 1 - e^{-x/a}, \ x \geq 0, \quad u(w) = \sqrt{w}$$  \hspace{1cm} (3)$$

Here the expected output is $Ex = a$, so higher effort increases expected output and leads to a more favorable distribution in the sense of first order stochastic dominance. The distribution satisfies MLRP. The utility function implies constant relative risk aversion ($-wu''/u' = \text{const}$).

It turns out that the marginal contractual incentives for effort in this case are constant and independent of effort, i.e. $\frac{\partial}{\partial a} m(a, w(v)) = 0$. So from (1)
we have here (for \( a = a(v) \))

\[
\frac{v^2}{C''(a)} \frac{\partial}{\partial v} m(a, w(v)) = \frac{vC''(a)}{C'(a)} a'(v) - 1
\]  

(4)

Hence we see that if the equilibrium marginal cost \( C'(a(v)) \) is inelastic (as a function of \( v \)) then marginal contractual incentives will be reduced as the level of enforceability \( v \) increases. If at the same time effort increases with higher \( v \), then clearly effort and contractual incentives will move in opposite directions.

It can be shown (see the appendix) that this will indeed be the case if the cost function exhibits inelastic marginal costs (\( aC''(a)/C'(a) \leq 1 \)) and moreover \( aC'''(a)/C''(a) > -3 \). (This holds e.g. for quadratic costs; \( C(a) = ca^2 \)). Thus we provide a set of conditions where effort increases while the incentives for effort generated by the contract decrease. (A somewhat more general result is given in the appendix; see Lemma 2.)

**Proposition 1**  If functional forms satisfy (3), then effort and contractual incentives are negatively related if marginal effort costs are inelastic (\( aC''(a)/C'(a) \leq 1 \)) and \( aC'''(a)/C''(a) > -3 \).

The intuition is as follows. Improved enforceability increases the agent’s incentives to exert effort (other things equal), but it also increases the principal’s wage costs per unit of effort (since the probability that the principal actually has to pay as promised increases). Now, even though the principal finds it optimal to induce higher effort when \( v \) increases, she will make a trade-off between the benefits from higher effort and the expected wage costs from higher \( v \). She may thus reduce these wage costs by providing lower-powered incentives. In other words, improved enforcement may crowd out contractual incentives.

Note that this type of crowding out appears when effort costs are inelastic, meaning that the agent has a high responsiveness to incentives. The reason is that improved enforcement increases effort and thus wage costs per unit effort.
to such an extent that the principal finds it optimal to reduce contractual incentives.

3.2 Limited liability

We will now show that similar results can be obtained under risk neutrality and limited liability. We assume from now on that the agent is risk neutral in the sense that \( u(w) = w \), but that he is protected by limited liability so that \( w(x) \geq 0 \). We also assume that the principal has limited means so that \( w(x) \leq x \). Hence, it is assumed that the principal cannot commit to pay wages above the agent’s value added. This constraint resembles Innes (1990) who in a financial contracting setting assumes that the investor’s (principal’s) liability is limited to her investment in the agent. Finally, it is convenient here to specify that output has support \( X = [\underline{x}, \overline{x}] \).

Now, the game proceeds as in the previous section, but under risk neutrality, the agent’s expected payoff is simply: 

\[
\int_{\underline{x}}^{\overline{x}} v(\beta(x)) f(x, a) dx - C(a),
\]

yielding a first order condition for effort as follows:

\[
\int_{\underline{x}}^{\overline{x}} v(\beta(x)) f_a(x, a) dx - C'(a) = 0 \quad (IC')
\]

In stage 1, the principal maximizes her payoff, which is

\[
\int_{\underline{x}}^{\overline{x}} (x - v(\beta(x))) f(x, a) dx - s,
\]

subject to incentive (IC’), participation (IR) and limited liability constraints:

\[
s + \int_{\underline{x}}^{\overline{x}} v(\beta(x)) f(x, a) dx - C(a) \geq U_o \quad (IR)
\]

\[
x \geq w(x) = s + \beta(x) \geq 0
\]

Mainly to simplify notation, we will assume \( \underline{x} = 0 \) and hence that the fixed
salary must be $s = 0$. By the same argument as in Innes (1990), it then follows that the optimal wage scheme pays the minimal wage for outcomes below some threshold, and the maximal wage for outcomes above that threshold ($\beta(x) = 0$ for $x < x_0$ and $\beta(x) = x$ for $x > x_0$). It is well known that the discontinuity of this scheme is problematic, and for that reason one requires continuity and monotonicity. The optimal such scheme also has a threshold (say $x_0$) and pays $\beta(x) = 0$ for $x \leq x_0$ and $\beta(x) = x - x_0$ for $x > x_0$. In the following we will focus on this kind of (constrained optimal) incentive scheme. Since the expected marginal payoff from exerting extra effort is zero as long as output is below $x_0$, it is clear that the higher is the threshold $x_0$, the lower is the incentive intensity of the contract.

Given that the principal cannot extract rent from the agent through the fixed salary component, the IR constraint will not bind unless the agent’s reservation utility $U_o$ is ‘large’. Mainly to simplify notation we will assume here that $U_o = 0$ and hence that this constraint is not binding.

Given the form of the incentive scheme, the expected payment for the agent is now

$$v \int_x^{x_0} \beta(x) f(x,a) dx = v \int_{x_0}^x (x - x_0) f(x,a) dx = v \int_{x_0}^x G(x,a) dx,$$

where the expression in the last integral follows from integration by parts, and where $G(x,a) = \Pr(outcome > x|a) = 1 - F(x,a)$. By a similar calculation the principal’s expected payoff can be written as

$$\int_x^{x_0} x f(x,a) dx - v \int_x^{x_0} \beta(x) f(x,a) dx = \int_x^{x_0} G(x,a) dx - v \int_{x_0}^x G(x,a) dx \quad (5)$$

The principal’s problem is now (for a given $v$) to choose $x_0, a$ to maximize this payoff subject to the agent’s incentive constraint.

We will focus on cases where higher $v$ is valuable for the principal.\footnote{If the principal can influence the verification probability $v$, e.g. by making costly}
that a higher $v$ is beneficial for the principal because it strengthens the agent’s incentives, but is on the other hand costly because it increases the total expected payments (and therefore the rent) to the agent. It turns out that a higher $v$ is valuable if $G_a(x, a) > 0$, meaning that more effort yields a shift to a distribution that is more favorable in the sense of first order stochastic dominance. As is well known, this is implied by MLRP.

Again, we analyze the following question: what happens to the optimal effort ($a$) and incentive scheme (represented by $x_0$) when $v$ varies? Comparative statics yields the following

**Lemma 1** If (in addition to MLRP) we have

$$\frac{\partial}{\partial a} \int_{x_0}^x \frac{G_a(x, a)}{G_a(x_0, a)} dx > 0$$

then $a'(v) > 0$.

As noted before, a improved enforcement increases the agent’s incentives to exert effort (other things equal), but it also increases the principal’s wage costs per unit of effort. The proposition gives conditions under which the first effect dominates in the sense that the principal finds it optimal to induce higher effort when enforceability increases. But the principal may still want to mitigate the latter effect, that is to reduce wage costs by providing lower-powered incentives. The next result shows that this is indeed what will occur, under some conditions. The following conditions turn out to be sufficient:

$$G_{aa}(x, a) < 0, \quad \frac{\partial}{\partial a} \frac{G_{aa}(x, a)}{G_a(x, a)} \leq 0 \quad \text{and} \quad \frac{\partial}{\partial x} \frac{G_{aa}(x, a)}{G_a(x, a)} > 0$$

**Proposition 2** Suppose that $C''(a) \geq 0$ and that $G(x, a)$ in addition to the assumptions in Lemma 1 satisfies (7). Then both effort and the threshold investments (say $K(v)$) in better contract specifications or performance metrics, we will have $\partial L/\partial v = K'(v)$ in optimum and thus $\partial L/\partial v > 0$ for the relevant level $v$. 

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for the incentive scheme increase with higher enforceability ($a'(v) > 0$ and $x_0'(v) > 0$), hence higher effort is then associated with lower-powered contractual incentives.

An example that satisfies all assumptions is $G(x, a) = \Pr(outcome > x) = 1 - x^a, 0 \leq x \leq 1$, (see the appendix).

The proposition demonstrates that higher effort may be associated with lower-powered contractual incentives (higher $x_0$), and the other way around, even if there is no motivation-crowding-out.

4 Concluding remarks

We offer a simple model where contractual monetary incentives and effort are negatively related even if there is no crowding out of non-monetary motivation. The idea is simple: Improved enforcement induces higher effort, but increases the principal’s expected wage costs, which can be mitigated by lower-powered incentives. Or: Weaker enforcement induces lower effort, which can be mitigated by higher-powered incentives.

Our model is not an alternative to the behavioral models on crowding out, but a complement. In contrast to (parts of) the crowding out literature, we do not offer a negative causal relationship between incentives and effort. Instead we identify a spurious relationship where improved contract enforcement increases effort but "crowd out" contractual incentives. Total monetary incentives, which is the product of the enforcement probability and contractual incentives, are positively related to effort, but since the enforcement probability does not show up in the incentive contract, it appears that incentives and effort are negatively related. The empirical implication is clear: When observing a negative relationship between performance pay and effort, one has to control for the probability that the relevant incentive contracts are actually enforced. If not, one may wrongfully infer that monetary
incentives crowd out non-monetary motivation.

Appendix

Proof of Proposition 1.

From the Lagrangian \( L = V + \lambda(U - U_o) + \mu U_a \), we obtain the following conditions for optimal bonuses \( \beta(x) \), or equivalently payments \( w(x) = s + \beta(x) \):

\[
0 = -vf(x,a) + \lambda vf(x,a)u'(w(x)) + \mu f_a(x,a)u'(w(x)),
\]

and for the optimal fixed payment \( s \):

\[
0 = -1 + \lambda \left( v \int f(x,a)u'(w(x))dx + (1 - v)u'(s) \right) + \mu \int f_a(x,a)u'(w(x))dx.
\]

The first is equivalent to \( \frac{1}{u'} = \lambda + \mu \frac{f_a}{f} \), and substituting from the first into the second we get \( \lambda u' = 1 \). This proves (W).

For utility \( u(w) = \sqrt{w} \) we have \( 1/u' = 2u \), hence the conditions for optimal payments are

\[
2u(w(x)) = \lambda + \mu \frac{f_a(x,a)}{f(x,a)} \equiv \lambda + \mu h(x,a), \quad 2u(w(s)) = \lambda \quad (8)
\]

where \( h(x,a) = \frac{f_a(x,a)}{f(x,a)} \) denotes the likelihood ratio.

Proposition 1 now follows from the lemma below. To state the lemma define

\[
M(a) = \int f_a(x,a)h(x,a)dx \quad (9)
\]

\[
M_1(a) = \int f_{aa}(x,a)h(x,a)dx \quad (10)
\]

\[
N(a) = \int f_a(x,a)h^2(x,a)dx \quad (11)
\]
Define also

\[
p(a) = 2(U_0 + C(a))C'(a) - \int x f_a(x,a) dx
\]

\[
q(a) = 2 \left[ \frac{N(a) - 2M_1(a)}{2M(a)} + \frac{C''(a)}{C'(a)} \right] \frac{C'(a)^2}{M(a)}
\]

Then we have

**Lemma 2** Assume \( u(w) = w^{-1/2} \). Then optimal effort satisfies \( p(a) + q(a)/v = 0 \). If \( q(a) > 0 \) then \( a'(v) > 0 \). If in addition condition (12) below holds, then \( \frac{\partial m}{\partial v} < 0 \).

\[
\left[ \frac{C''(a)}{C'(a)} - \frac{M_1(a)}{M(a)} \right] \frac{q(a)/v}{p'(a) + q'(a)/v} - 1 < 0 \tag{12}
\]

As we will show below, the LHS of (12) coincides with (2). Consider now Proposition 1. For \( F(x,a) = 1 - e^{-x/a} \) it is straightforward to verify (see below) that we have \( M(a) = 1/a^2 \), \( M_1(a) = 0 \), \( N(a) = 2/a^3 \) and \( \int x f_a(x,a) dx = 1 \), and hence that

\[
q(a) = 2 \left[ \frac{1}{a} + \frac{C''(a)}{C'(a)} \right] C'(a)^2 a^2
\]

For this distribution, condition (12) in the lemma is thus

\[
\left[ \frac{a C''(a)}{C'(a)} \right] \frac{q(a)/v}{ap'(a) + aq'(a)/v} < 1
\]

Since \( p'(a) > 0 \), we see that for inelastic marginal costs this condition holds if \( q(a) \leq aq'(a) \). This holds if \( \frac{a C''(a)}{C'(a)} \geq -3 \) (see below), proving Proposition 1.
Proof of the lemma. Consider first the agent’s marginal contractual incentive $m(a, w)$, where payments $w()$ are optimal, and thus given by (8) for the optimal action $a = a^*$, say. We then have

$$m(a, w) = \int f_a(x, a)u(w(x))dx = \int f_a(x, a)\frac{\lambda + \mu h(x, a^*)}{2}dx \quad (13)$$

$$= \int f_a(x, a)h(x, a^*)dx \frac{\mu}{2} \equiv M(a, a^*)\frac{\mu}{2} \quad (14)$$

where $M(a, a^*)$ is (with a slight abuse of notation) defined by the identity, and the third equality follows from $\int f_a = 0$ (since $\int f = 1$).

Note that the agent’s choice problem is concave if $vm_a(a, w) - C''(a) \leq 0$, which holds if $M_a(a, a^*) \leq 0$ and $C'' \geq 0$, and that the optimal choice of effort is then given by the FOC $vm(a, w) = C'(a)$. In equilibrium we have $a = a^*$ and thus

$$C'(a) = v \int f_a(x, a)u(w(x))dx = v\frac{\mu}{2} M(a, a) \equiv v\frac{\mu}{2} M(a) \quad (15)$$

Note also from IR (which will be binding) and (8) that we have

$$U_o + C(a) = v \int f(x, a)u(w(x))dx + (1 - v)u(s)$$

$$= v \int f(x, a) [\lambda + \mu h(x, a)] /2dx + (1 - v)\lambda /2 = \lambda /2 \quad (16)$$

where the last equality follows from the fact that $\int fh = \int f \frac{f_v}{f} = \int f_a = 0$. Hence we see that $\lambda = 2(U_o + C(a))$.

To characterize the optimal effort for the principal, consider

$$L_a = V_a + \lambda U_a + \mu U_{aa}$$

$$= \int f_a(x, a) [x - vw(x)] dx + 0 + \mu (v \int f_{aa}(x, a)u(w(x))dx - C''(a))$$

$$= e(a) - v \int f_a(x, a)w(x)dx + \mu (vm_a(a, w) - C''(a)) \quad (17)$$
where we have defined $e(a) = \int xf_a(x, a)dx$ as the marginal value of effort on output.

Consider the second term in (17). Since $u = \sqrt{w}$ we have $w = u^2$, and substituting from (8) we can write

$$
\int f_a(x, a)w(x)dx = \int f_a(x, a) \left[ \frac{\lambda + \mu h(x, a)}{2} \right]^2 dx = \int f_a(x, a) \left[ \lambda^2 + 2\lambda \mu h(x, a) + \mu^2 h^2(x, a) \right] dx/4 = \lambda \frac{\mu}{2} M(a) + \frac{\mu^2}{4} N(a)
$$

(18)

where the last equality follows from $\int f_a = 0$ and the definitions of $M(a)$ and $N(a)$, see (9) and (11).

We see from (13) and (10) that we (in equilibrium) have $m_a(a, w) = M_1(a)\mu/2$ and hence that (17) can be written as

$$
L_a = e(a) - v \left( \lambda \frac{\mu}{2} M(a) + \frac{\mu^2}{4} N(a) \right) + \mu \left( v \frac{\mu}{2} M_1(a) - C''(a) \right)
$$

Substituting for $\mu$ from (15) and for $\lambda$ from (16) we obtain the following condition for optimal effort

$$
0 = L_a = e(a) - v \left( \lambda C'(a) + \frac{C'(a) \mu}{M(a)} 2 \mu N(a) \right) + \left( \frac{C'(a)}{M(a)} M_1(a) - C''(a) \right) \frac{C'(a) \mu}{M(a) v}
$$

where the last equality follows from the definitions of $p(a), q(a)$ and $e(a) = \int xf_a$.

This shows that optimal effort is given by $p(a) + q(a)\frac{1}{v} = 0$, as stated in the lemma, and that $a'(v) = \frac{q(a)/v^2}{p(a) + q(a)/v}$. Concavity of the principal’s optimization w.r.t. effort requires $p'(a) + q'(a)/v > 0$, and hence we have
that and in effort, since for the distribution

Finally note that this shows that and likelihood ratio the distribution proof.

For completeness we finally verify the assertions stated above regarding the distribution \( F(x, a) = 1 - e^{-x/a} \). We have here density \( f(x, a) = \frac{1}{a} e^{-x/a} \) and likelihood ratio \( h(x, a) = \frac{f_a(x, a)}{f(x, a)} = \frac{1}{a} (\frac{x}{a} - 1) \). Hence

\[
M(a, a^*) = \int f_a(x, a) h(x, a^*) dx = \int_0^\infty \frac{1}{a^2} e^{-x/a} (\frac{x}{a} - 1) \frac{1}{a^*} (\frac{x}{a^*} - 1) dx
\]

\[
= \frac{1}{aa^*} \int_0^\infty e^{-y}(y-1)(\frac{a}{a^*} - 1) dy = \frac{1}{(a^*)^2}
\]

This shows that \( M(a) = M(a, a) = 1/a^2 \) and \( M_1(a) = M_a(a, a^* = a) = 0 \). We further have

\[
N(a) = \int f_a(x, a) h^2(x, a) dx = \int_0^\infty \frac{1}{a^4} e^{-x/a} (\frac{x}{a} - 1)^3 dx = \frac{1}{a^3} \int_0^\infty e^{-y}(y-1)^3 dy = \frac{2}{a^3}
\]

Finally note that \( q(a) = 2a \psi(a) \), with \( \psi(a) = (C')^2 + aC''C' \) and hence that \( aq'(a) = 2a \psi'(a) + 2a^2 \psi''(a) \geq q(a) \) if \( \psi'(a) \geq 0 \). We have \( \psi'(a) = 3C'C'' + aC'''C' + aC''C'' > 0 \) certainly if \( aC'''/C'' \geq -3 \). This verifies the stated assertions.

**Remark.** As another application of the Lemma, one can show that effort and contractual incentives move in opposite directions \( (a'(v) > 0, \frac{\partial m}{\partial v} < 0) \) for the distribution \( F(x, a) = x^a, x \in [0, 1] \) if \( C'(a) \) is sufficiently inelastic and \( v \) is sufficiently large (close to 1). For this distribution one finds \( M(a) = \frac{1}{a^2}, N(a) = -\frac{2}{a^3}, M_1(a) = -\frac{2}{a^4} \) (and hence marginal incentives are decreasing in effort, since \( m_a = M_1 \mu/2 < 0 \)). Assuming \( C'(a) = k = const \), we then find \( q(a) = \frac{N(a) - 2M_1(a)}{M(a)} \frac{k^2}{M(a)} = 2ak^2 \) and \( p(a) = 2(U_0 + ka)k - \frac{1}{(a+1)^2} \), and thus

\[
\left[ \frac{C''(a)}{C'(a)} - \frac{M_1(a)}{M(a)} \right] \frac{q(a)/v}{p'(a)+q(a)/v} = \left[ \frac{2}{a} \right] \frac{2a^2 k^2}{2k^2 + \frac{2a^2 k^2}{(a+1)^2} + 2k^2/v}
\]

20
\[
\frac{1}{v + \frac{1}{(a+1)^2}} \rightarrow 2 \frac{1}{2+ \frac{1}{(a+1)^2}} < 1 \quad \text{as} \; v \rightarrow 1
\]

This shows that the condition in the Lemma is fulfilled for \( v \) close to 1.

**Proof of Lemma 1**

The principal chooses \( x_0, a \) to maximize her payoff (5) subject to the agent’s incentive constraint, which here takes the form

\[
v \int_{x_0}^{\bar{x}} G_a(x, a) \, dx - C'(a) = 0
\]  

(19)

The Lagrangian for this problem is

\[
L = \int_{\underline{x}}^{\bar{x}} G(x, a) \, dx - v \int_{x_0}^{\bar{x}} G(x, a) \, dx + \mu \left[ v \int_{x_0}^{\bar{x}} G_a(x, a) \, dx - C'(a) \right]
\]  

(20)

As noted we focus on cases where higher \( v \) is valuable for the principal, i.e. where \( \frac{\partial L}{\partial v} > 0 \). Since optimization with respect to the threshold parameter \( x_0 \) yields \( vG(x_0, a) - v\mu G_a(x_0, a) = 0 \) and hence \( \mu = \frac{G(x_0, a)}{G_a(x_0, a)} \), we have

\[
\frac{\partial L}{\partial v} = - \int_{x_0}^{\bar{x}} G(x, a) \, dx + \mu \int_{x_0}^{\bar{x}} G_a(x, a) \, dx = \int_{x_0}^{\bar{x}} \left[ \frac{G(x_0, a)}{G_a(x_0, a)} - \frac{G(x, a)}{G_a(x, a)} \right] G_a(x, a) \, dx
\]  

(21)

We see that we will have \( \frac{\partial L}{\partial v} > 0 \) if \( G_a(x, a) > 0 \) and the ratio \( \frac{G(x, a)}{G_a(x, a)} \) is decreasing in \( x \). Both properties follow from MLRP; we demonstrate the latter below (at the end of this proof).

Consider now the Lagrangian (20) and write the constraint (19) as

\[
H(x_0, a, v) \equiv v \int_{x_0}^{\bar{x}} G_a(x, a) \, dx - C'(a) = 0
\]  

(22)

The FOCs for optimal choices are \( L_{x_0} = L_a = H = 0 \). (Subscripts denote
Differentiation of these conditions yields

\[
\begin{bmatrix}
L_{xx} & L_{xa} & H_x & L_{x0} \\
L_{ax} & L_{aa} & H_a & L_{a0} \\
H_{x0} & H_a & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_0'(v) \\
a'(v) \\
\mu'(v)
\end{bmatrix} =
\begin{bmatrix}
-L_{x0v} \\
-L_{av} \\
-H_v
\end{bmatrix}
\]

and hence the standard comparative statics formulae

\[
x_0'(v) = \frac{1}{D} \begin{vmatrix}
-L_{x0v} & L_{x0a} & H_x & L_{x0} \\
-L_{av} & L_{aa} & H_a & L_{a0} \\
-H_v & H_a & 0 & 0
\end{vmatrix} = \frac{1}{D} \left[H_a^2 L_{x0x0} - H_a H_v L_{av} - H_a H_v L_{a0x0} + H_v H_{x0} L_{aa}\right]
\]

\[
a'(v) = \frac{1}{D} \begin{vmatrix}
L_{x0x0} & -L_{x0v} & H_x & L_{x0} \\
L_{ax} & -L_{av} & H_a & L_{a0} \\
H_{x0} & -H_v & 0 & 0
\end{vmatrix} = \frac{1}{D} \left[H_a^2 L_{av} - H_v H_{x0} L_{ax0} - H_a H_{x0} L_{ax0} + H_a H_v L_{x0x0}\right]
\]

where

\[
D = \begin{vmatrix}
L_{x0x0} & L_{x0a} & H_x & L_{x0} \\
L_{ax} & L_{aa} & H_a & L_{a0} \\
H_{x0} & H_a & 0 & 0
\end{vmatrix} = -L_{x0x0} H_a^2 + 2L_{ax} H_a H_{x0} - L_{aa} H_v^2 > 0 \quad \text{(SOC)}
\]

From FOC we have \(0 = L_{x0} = vG(x_0, a) - \mu G_a(x_0, a)\) and hence

\[
L_{x0} = G(x_0, a) - \mu G_a(x_0, a) = 0 \quad (23)
\]

Hence we can write

\[
x_0'(v)D = -H_a H_{x0} L_{av} - H_a H_v L_{ax0} + H_v H_{x0} L_{aa} \quad (24)
\]

\[
a'(v)D = H_a^2 L_{av} - H_v H_{x0} L_{ax0} + H_a H_v L_{x0x0} \quad (25)
\]
Writing \( g(x, a) = G_x(x, a) \) and using (23) we have

\[
L_{x_0,x_0}/v = g(x_0, a) - \mu g_a(x_0, a) = g(x_0, a) - \frac{G(x_0, a)}{G_a(x_0, a)} g_a(x_0, a) < 0
\]

where the inequality holds because we have assumed \( G_a > 0 \) and it follows from MLRP (as shown below) that \( \frac{d}{dx} \frac{G_a}{G} = \frac{1}{G^2} (g_a G - G_a g) > 0 \).

From (22), \( G_a > 0 \) and the SOC for the agent we have

\[
H_{x_0} = -v G_a(x_0, a) < 0, \quad H_v = \int_{x_0}^{x} G_a(x, a) dx > 0, \quad H_a = v \int_{x_0}^{x} G_{aa}(x, a) dx - C''(a) < 0
\]

(26)

These inequalities imply \( H_a H_v L_{x_0,x_0} > 0 \), and we thus have from (25): \( a'(v) D > [H_{x_0} L_{av} - H_v L_{ax_0}] H_{x_0} \).

Since \( H_{x_0} = -v G_a < 0 \) we then have \( a'(v) > 0 \) if \( H_{x_0} L_{av} - H_v L_{ax_0} < 0 \).

To show that this condition implying \( a'(v) > 0 \) is satisfied, consider

\[
H_{x_0} L_{av} - H_v L_{ax_0} = -v G_a(x_0, a) \left[ - \int_{x_0}^{x} G_a(x, a) dx + \mu \int_{x_0}^{x} G_{aa}(x, a) dx \right]
- \left( \int_{x_0}^{x} G_a(x, a) dx \right) \left[ v G_a(x_0, a) - \mu v G_{aa}(x_0, a) \right]
= \mu v \left[ -G_a(x_0, a) \int_{x_0}^{x} G_{aa}(x, a) dx + G_{aa}(x_0, a) \int_{x_0}^{x} G_a(x, a) dx \right]
= -\mu v G_a^2(x_0, a) \left[ \frac{\partial}{\partial a} \int_{x_0}^{x} \frac{G_a(x, a)}{G_a(x_0, a)} dx \right] < 0
\]

The last inequality follows from the assumption (6) and proves that \( a'(v) > 0 \).

It remains to verify the assertion – stated after (21) – that MLRP implies that the ratio \( \frac{G(x, a)}{G_a(x, a)} \) is decreasing in \( x \). To this end consider

\[
\frac{\partial}{\partial x} \frac{G_a}{G} = \frac{1}{G^2} (g_a G - G_a g) = \frac{g}{G} \left( \frac{g_a - G_a}{G} \right)
\]

(27)

The derivative is positive, and the proof is thus complete, if the last paren-
thesis is positive. Note that

\[ \frac{G_a(x, a)}{G(x, a)} = \frac{\partial}{\partial a} \int_x^x g(x', a) dx' G(x, a) = \int_x^x \frac{g_a(x', a)}{g(x', a)} \frac{g(x', a)}{G(x, a)} dx' \leq \frac{g_a(x, a)}{g(x, a)} \cdot 1 \]

where the inequality follows by MLRP \( \frac{g_a(x, a)}{g(x, a)} = \frac{f_a(x, a)}{f(x, a)} \) increasing. Hence the derivative in (27) is positive, and this completes the proof.

**Proof of Proposition 2**

First note that \( G_{aa} < 0 \) implies \( L_{av} = -\int_{x_0}^x G_a(x, a) dx + \mu \int_{x_0}^x G_{aa}(x, a) dx < 0 \), and hence from (26) that \( H_a H_{x_0} L_{av} < 0 \). We then have from (24):

\[ x'_0(v)D = -H_a H_{x_0} L_{av} - H_a H_{x_0} L_{ax_0} + H_v H_{x_0} L_{aa} > [-H_a L_{ax_0} + H_{x_0} L_{aa}] H_v \]

(28)

Consider \([-H_a L_{ax_0} + H_{x_0} L_{aa}] \). Since \( H_a < v \int_{x_0}^x G_{aa}(x, a) dx \) by (26), and since \( G_{aa} < 0 \) implies \( L_{ax_0} = vG_a(x_0, a) - \mu vG_{aa}(x_0, a) > 0 \), we have

\[ -H_a L_{ax_0} + H_{x_0} L_{aa} > - \left( v \int_{x_0}^x G_{aa}(x, a) dx \right) \left[ G_a(x_0, a) - \mu G_{aa}(x_0, a) \right] v + (-vG_a(x_0, a)) L_{aa} \]

\[ = vG_a(x_0, a) \left[ - \int_{x_0}^x G_{aa}(x, a) dx \left[ 1 - \frac{G_{aa}(x_0, a)}{G_a(x_0, a)} \right] \right] v - L_{aa} \]  

(29)

Consider \( L_{aa} \). Since \( G_{aa} < 0 \) and \( C'''(a) \geq 0 \) we have

\[ L_{aa} = \int_{x_0}^x G_{aa}(x, a) dx - v \int_{x_0}^x G_{aa}(x, a) dx + \mu \left[ v \int_{x_0}^x G_{aaa}(x, a) dx - C'''(a) \right] \]

\[ < \int_{x_0}^x G_{aa}(x, a) \left[ 1 - v + \mu v \frac{G_{aaa}(x, a)}{G_{aa}(x, a)} \right] dx \]

24
Hence from (29) we now have

$$-H_a L_{ax_0} + H_{x_0} L_{aa} \over v G_a(x_0, a) > - \int_{x_0}^x G_{aa}(x, a) \left[ 1 + \mu_v \left( \frac{G_{aaa}(x, a)}{G_{aa}(x, a)} - \frac{G_{aa}(x_0, a)}{G_a(x_0, a)} \right) \right] dx > 0$$

(30)

where the last inequality will be shown to follow from (7). From (28) and the fact that $H_v > 0$ we then see that $x_0'(v) > 0$.

To show the last inequality in (30), note that the assumptions in (7) imply

$$\frac{\partial}{\partial a} \frac{G_{aa}(x, a)}{G_a(x, a)} = \frac{G_{aa}(x, a)}{G_a(x, a)} \left( \frac{G_{aaa}(x, a)}{G_{aa}(x, a)} - \frac{G_{aa}(x_0, a)}{G_a(x_0, a)} \right) \leq 0$$

and $\frac{G_{aa}(x, a)}{G_a(x, a)} > \frac{G_{aa}(x_0, a)}{G_a(x_0, a)}$ when $x > x_0$. These inequalities in turn imply

$$\frac{G_{aaa}(x, a)}{G_{aa}(x, a)} > \frac{G_{aa}(x_0, a)}{G_a(x_0, a)}$$

when $x > x_0$

This implies that the expression in (30) is positive, and hence completes the proof that $x_0'(v) > 0$.

To illustrate the assumptions stated in Proposition 3, we finally show that they are all satisfied by $G(x, a) = 1 - x^a$, $0 \leq x \leq 1$. For this distribution we have

$$G_a(x, a) = -x^a \ln x > 0$$
$$G_{ax}(x, a) = -ax^{a-1} = -f(x, a)$$
$$G_{xa}(x, a) = -f_a(x, a) = -x^{a-1} (a \ln x + 1)$$

Hence $\frac{f_a(x, a)}{f(x, a)} = \ln x + 1/a$ is increasing in $x$, so MLRP holds. Moreover, we also have

$$\frac{\partial}{\partial a} \int_{x_0}^x \frac{G_a(x, a)}{G_a(x_0, a)} dx = \frac{\partial}{\partial a} \int_{x_0}^1 \frac{x^a \ln x}{x_0^a \ln x_0} dx = \int_{x_0}^1 \frac{(x/a)^a \ln x}{x_0} \ln x_0 dx > 0$$

hence the condition stated in Lemma 1 holds.

Next note that

$$G_{aa}(x, a) = -\frac{d}{da} x^a \ln x = -x^a (\ln x)^2 = G_a(x, a) \ln x < 0$$
and hence that \( \frac{G_{a}(x,a)}{G_{a}(x,a)} = \ln x \). The additional assumptions (7) in Proposition 2 are therefore also satisfied.

References


